

Rotation and Tiling Groups, Genus 2-13

S. Allen Broughton, Robert M. Dirks*,
Maria T. Slougher*, C. Ryan Vinroot*

February 15, 2001

The tables that follow give for each triple of a genus σ a group G and branching triple (l, m, n) the number of isometric equivalence classes of actions separated into kaleidoscopic and non-kaleidoscopic classes. Presentations of all groups and the corresponding generating vectors for each action are available in electronic form at the website [1].

Notation

| | |
|----------|---|
| σ | genus of the surface |
| $ G $ | order of the rotation group G |
| Group | Z_n : cyclic group of order n |
| | $G(g, n)$: n 'th small group of order g in the Magma data base |
| #Kal | number of kaleidoscopic actions yielding a tiling and, hence, a tiling group G^* |
| #non-Kal | number of non-kaleidoscopic actions |
| total | #Kal + #non-Kal |
| Type | C = cyclic group, A2 = 2 generator, non-cyclic abelian group |
| | p-NA = non-abelian p -group |
| | S-NA-NP = non-abelian solvable group, but not a p -group |
| | NS = non-solvable |

* Author supported by NSF grant #DMS-9619714

Table 1 Rotation and Tiling Groups, Genus 2-13 - part 1

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|-------------|------------------|------|----------|-------|---------|
| 2 | 5 | (5, 5, 5) | Z_5 | 1 | 0 | 1 | C |
| 2 | 6 | (3, 6, 6) | Z_6 | 1 | 0 | 1 | C |
| 2 | 8 | (2, 8, 8) | Z_8 | 1 | 0 | 1 | C |
| 2 | 8 | (4, 4, 4) | $G(8, 4)$ | 1 | 0 | 1 | p-NA |
| 2 | 10 | (2, 5, 10) | Z_{10} | 1 | 0 | 1 | C |
| 2 | 12 | (2, 6, 6) | $Z_2 \times Z_6$ | 1 | 0 | 1 | A2 |
| 2 | 12 | (3, 4, 4) | $G(12, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 2 | 16 | (2, 4, 8) | $G(16, 8)$ | 1 | 0 | 1 | p-NA |
| 2 | 24 | (2, 4, 6) | $G(24, 8)$ | 1 | 0 | 1 | S-NA-NP |
| 2 | 24 | (3, 3, 4) | $G(24, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 2 | 48 | (2, 3, 8) | $G(48, 29)$ | 1 | 0 | 1 | S-NA-NP |
| | | | | | | | |
| 3 | 7 | (7, 7, 7) | Z_7 | 2 | 0 | 2 | C |
| 3 | 8 | (4, 8, 8) | Z_8 | 2 | 0 | 2 | C |
| 3 | 9 | (3, 9, 9) | Z_9 | 1 | 0 | 1 | C |
| 3 | 12 | (2, 12, 12) | Z_{12} | 1 | 0 | 1 | C |
| 3 | 12 | (3, 4, 12) | Z_{12} | 1 | 0 | 1 | C |
| 3 | 12 | (4, 4, 6) | $G(12, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 14 | (2, 7, 14) | Z_{14} | 1 | 0 | 1 | C |
| 3 | 16 | (2, 8, 8) | $Z_2 \times Z_8$ | 1 | 0 | 1 | A2 |
| 3 | 16 | (2, 8, 8) | $G(16, 6)$ | 1 | 0 | 1 | p-NA |
| 3 | 16 | (4, 4, 4) | $Z_4 \times Z_4$ | 1 | 0 | 1 | A2 |
| 3 | 16 | (4, 4, 4) | $G(16, 4)$ | 1 | 0 | 1 | p-NA |
| 3 | 21 | (3, 3, 7) | $G(21, 1)$ | 0 | 1 | 1 | S-NA-NP |
| 3 | 24 | (2, 4, 12) | $G(24, 5)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 24 | (2, 6, 6) | $G(24, 13)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 24 | (3, 3, 6) | $G(24, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 24 | (3, 4, 4) | $G(24, 12)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 32 | (2, 4, 8) | $G(32, 9)$ | 1 | 0 | 1 | p-NA |
| 3 | 32 | (2, 4, 8) | $G(32, 11)$ | 1 | 0 | 1 | p-NA |
| 3 | 48 | (2, 3, 12) | $G(48, 33)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 48 | (2, 4, 6) | $G(48, 48)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 48 | (3, 3, 4) | $G(48, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 96 | (2, 3, 8) | $G(96, 64)$ | 1 | 0 | 1 | S-NA-NP |
| 3 | 168 | (2, 3, 7) | $G(168, 42)$ | 1 | 0 | 1 | NS |

Table 2 Rotation and Tiling Groups, Genus 2-13 - part 2

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 4 | 9 | (9, 9, 9) | Z_9 | 1 | 0 | 1 | C |
| 4 | 10 | (5, 10, 10) | Z_{10} | 2 | 0 | 2 | C |
| 4 | 12 | (3, 12, 12) | Z_{12} | 1 | 0 | 1 | C |
| 4 | 12 | (4, 6, 12) | Z_{12} | 1 | 0 | 1 | C |
| 4 | 12 | (6, 6, 6) | $Z_2 \times Z_6$ | 1 | 0 | 1 | A2 |
| 4 | 15 | (3, 5, 15) | Z_{15} | 1 | 0 | 1 | C |
| 4 | 16 | (2, 16, 16) | Z_{16} | 1 | 0 | 1 | C |
| 4 | 16 | (4, 4, 8) | $G(16, 9)$ | 1 | 0 | 1 | p-NA |
| 4 | 18 | (2, 9, 18) | Z_{18} | 1 | 0 | 1 | C |
| 4 | 18 | (3, 6, 6) | $Z_3 \times Z_6$ | 1 | 0 | 1 | A2 |
| 4 | 18 | (3, 6, 6) | $G(18, 3)$ | 2 | 0 | 2 | S-NA-NP |
| 4 | 20 | (2, 10, 10) | $Z_2 \times Z_{10}$ | 1 | 0 | 1 | A2 |
| 4 | 20 | (4, 4, 5) | $G(20, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 20 | (4, 4, 5) | $G(20, 3)$ | 0 | 1 | 1 | S-NA-NP |
| 4 | 24 | (2, 6, 12) | $G(24, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 24 | (3, 4, 6) | $G(24, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 32 | (2, 4, 16) | $G(32, 19)$ | 1 | 0 | 1 | p-NA |
| 4 | 36 | (2, 6, 6) | $G(36, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 36 | (2, 6, 6) | $G(36, 12)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 36 | (3, 3, 6) | $G(36, 11)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 36 | (3, 4, 4) | $G(36, 9)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 40 | (2, 4, 10) | $G(40, 8)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 60 | (2, 5, 5) | $G(60, 5)$ | 1 | 0 | 1 | NS |
| 4 | 72 | (2, 3, 12) | $G(72, 42)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 72 | (2, 4, 6) | $G(72, 40)$ | 1 | 0 | 1 | S-NA-NP |
| 4 | 120 | (2, 4, 5) | $G(120, 34)$ | 1 | 0 | 1 | NS |
| | | | | | | | |
| 5 | 11 | (11, 11, 11) | Z_{11} | 2 | 0 | 2 | C |
| 5 | 12 | (6, 12, 12) | Z_{12} | 1 | 0 | 1 | C |
| 5 | 15 | (3, 15, 15) | Z_{15} | 1 | 0 | 1 | C |
| 5 | 16 | (4, 8, 8) | $Z_2 \times Z_8$ | 1 | 0 | 1 | A2 |
| 5 | 16 | (4, 8, 8) | $G(16, 6)$ | 1 | 0 | 1 | p-NA |
| 5 | 20 | (2, 20, 20) | Z_{20} | 1 | 0 | 1 | C |
| 5 | 20 | (4, 4, 10) | $G(20, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 22 | (2, 11, 22) | Z_{22} | 1 | 0 | 1 | C |

Table 3 Rotation and Tiling Groups, Genus 2-13 - part 3

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 5 | 24 | (2, 12, 12) | $Z_2 \times Z_{12}$ | 1 | 0 | 1 | A2 |
| 5 | 24 | (3, 6, 6) | $G(24, 13)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 24 | (4, 4, 6) | $G(24, 7)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 30 | (2, 6, 15) | $G(30, 2)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 32 | (2, 8, 8) | $G(32, 5)$ | 1 | 0 | 1 | p-NA |
| 5 | 32 | (2, 8, 8) | $G(32, 7)$ | 1 | 0 | 1 | p-NA |
| 5 | 32 | (4, 4, 4) | $G(32, 2)$ | 1 | 0 | 1 | p-NA |
| 5 | 32 | (4, 4, 4) | $G(32, 6)$ | 1 | 0 | 1 | p-NA |
| 5 | 40 | (2, 4, 20) | $G(40, 5)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 48 | (2, 4, 12) | $G(48, 14)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 48 | (3, 4, 4) | $G(48, 30)$ | 2 | 0 | 2 | S-NA-NP |
| 5 | 60 | (3, 3, 5) | $G(60, 5)$ | 1 | 0 | 1 | NS |
| 5 | 64 | (2, 4, 8) | $G(64, 8)$ | 1 | 0 | 1 | p-NA |
| 5 | 64 | (2, 4, 8) | $G(64, 32)$ | 1 | 0 | 1 | p-NA |
| 5 | 80 | (2, 5, 5) | $G(80, 49)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 96 | (2, 4, 6) | $G(96, 195)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 96 | (3, 3, 4) | $G(96, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 120 | (2, 3, 10) | $G(120, 35)$ | 1 | 0 | 1 | NS |
| 5 | 160 | (2, 4, 5) | $G(160, 234)$ | 1 | 0 | 1 | S-NA-NP |
| 5 | 192 | (2, 3, 8) | $G(192, 181)$ | 1 | 0 | 1 | S-NA-NP |
| | | | | | | | |
| 6 | 13 | (13, 13, 13) | Z_{13} | 3 | 0 | 3 | C |
| 6 | 14 | (7, 14, 14) | Z_{14} | 3 | 0 | 3 | C |
| 6 | 15 | (5, 15, 15) | Z_{15} | 2 | 0 | 2 | C |
| 6 | 16 | (4, 16, 16) | Z_{16} | 1 | 0 | 1 | C |
| 6 | 18 | (3, 18, 18) | Z_{18} | 1 | 0 | 1 | C |
| 6 | 20 | (4, 5, 20) | Z_{20} | 1 | 0 | 1 | C |
| 6 | 21 | (3, 7, 21) | Z_{21} | 1 | 0 | 1 | C |
| 6 | 24 | (2, 24, 24) | Z_{24} | 1 | 0 | 1 | C |
| 6 | 24 | (3, 8, 8) | $G(24, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 24 | (4, 4, 12) | $G(24, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 24 | (4, 6, 6) | $G(24, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 24 | (4, 6, 6) | $G(24, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 25 | (5, 5, 5) | $Z_5 \times Z_5$ | 1 | 0 | 1 | A2 |
| 6 | 26 | (2, 13, 26) | Z_{26} | 1 | 0 | 1 | C |

Table 4 Rotation and Tiling Groups, Genus 2-13 - part 4

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 6 | 28 | (2, 14, 14) | $Z_2 \times Z_{14}$ | 1 | 0 | 1 | A2 |
| 6 | 28 | (4, 4, 7) | $G(28, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 30 | (2, 10, 15) | $G(30, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 36 | (2, 9, 9) | $G(36, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 39 | (3, 3, 13) | $G(39, 1)$ | 0 | 1 | 1 | S-NA-NP |
| 6 | 48 | (2, 4, 24) | $G(48, 6)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 48 | (2, 6, 8) | $G(48, 15)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 48 | (2, 6, 8) | $G(48, 29)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 50 | (2, 5, 10) | $G(50, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 56 | (2, 4, 14) | $G(56, 7)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 72 | (2, 4, 9) | $G(72, 15)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 75 | (3, 3, 5) | $G(75, 2)$ | 1 | 0 | 1 | S-NA-NP |
| 6 | 120 | (2, 4, 6) | $G(120, 34)$ | 1 | 0 | 1 | NS |
| 6 | 150 | (2, 3, 10) | $G(150, 5)$ | 1 | 0 | 1 | S-NA-NP |
| | | | | | | | |
| 7 | 15 | (15, 15, 15) | Z_{15} | 1 | 0 | 1 | C |
| 7 | 16 | (8, 16, 16) | Z_{16} | 3 | 0 | 3 | C |
| 7 | 18 | (6, 9, 18) | Z_{18} | 2 | 0 | 2 | C |
| 7 | 20 | (4, 10, 20) | Z_{20} | 1 | 0 | 1 | C |
| 7 | 21 | (3, 21, 21) | Z_{21} | 1 | 0 | 1 | C |
| 7 | 24 | (3, 8, 24) | Z_{24} | 1 | 0 | 1 | C |
| 7 | 24 | (4, 6, 12) | $Z_2 \times Z_{12}$ | 1 | 0 | 1 | A2 |
| 7 | 24 | (6, 6, 6) | $G(24, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 27 | (3, 9, 9) | $Z_3 \times Z_9$ | 1 | 0 | 1 | A2 |
| 7 | 27 | (3, 9, 9) | $G(27, 4)$ | 0 | 1 | 1 | p-NA |
| 7 | 28 | (2, 28, 28) | Z_{28} | 1 | 0 | 1 | C |
| 7 | 28 | (4, 4, 14) | $G(28, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 30 | (2, 15, 30) | Z_{30} | 1 | 0 | 1 | C |
| 7 | 32 | (2, 16, 16) | $Z_2 \times Z_{16}$ | 1 | 0 | 1 | A2 |
| 7 | 32 | (2, 16, 16) | $G(32, 17)$ | 1 | 0 | 1 | p-NA |
| 7 | 32 | (4, 4, 8) | $G(32, 10)$ | 1 | 0 | 1 | p-NA |
| 7 | 32 | (4, 4, 8) | $G(32, 11)$ | 1 | 0 | 1 | p-NA |
| 7 | 32 | (4, 4, 8) | $G(32, 13)$ | 1 | 0 | 1 | p-NA |
| 7 | 32 | (4, 4, 8) | $G(32, 14)$ | 1 | 0 | 1 | p-NA |
| 7 | 36 | (3, 4, 12) | $G(36, 6)$ | 1 | 0 | 1 | S-NA-NP |

Table 5 Rotation and Tiling Groups, Genus 2-13 - part 5

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 7 | 42 | (2, 6, 21) | $G(42, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 48 | (2, 6, 12) | $G(48, 33)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 48 | (3, 4, 6) | $G(48, 32)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 54 | (2, 6, 9) | $G(54, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 54 | (2, 6, 9) | $G(54, 6)$ | 0 | 1 | 1 | S-NA-NP |
| 7 | 56 | (2, 4, 28) | $G(56, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 56 | (2, 7, 7) | $G(56, 11)$ | 0 | 1 | 1 | S-NA-NP |
| 7 | 64 | (2, 4, 16) | $G(64, 38)$ | 1 | 0 | 1 | p-NA |
| 7 | 64 | (2, 4, 16) | $G(64, 41)$ | 1 | 0 | 1 | p-NA |
| 7 | 72 | (3, 3, 6) | $G(72, 25)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 144 | (2, 3, 12) | $G(144, 127)$ | 1 | 0 | 1 | S-NA-NP |
| 7 | 504 | (2, 3, 7) | $G(504, 156)$ | 1 | 0 | 1 | NS |
| | | | | | | | |
| 8 | 17 | (17, 17, 17) | Z_{17} | 3 | 0 | 3 | C |
| 8 | 18 | (9, 18, 18) | Z_{18} | 2 | 0 | 2 | C |
| 8 | 20 | (10, 10, 10) | $Z_2 \times Z_{10}$ | 1 | 0 | 1 | A2 |
| 8 | 20 | (5, 20, 20) | Z_{20} | 2 | 0 | 2 | C |
| 8 | 24 | (3, 24, 24) | Z_{24} | 1 | 0 | 1 | C |
| 8 | 24 | (4, 12, 12) | $G(24, 11)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 24 | (6, 6, 12) | $G(24, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 24 | (6, 8, 8) | $G(24, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 30 | (3, 10, 10) | $G(30, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 30 | (5, 6, 6) | $G(30, 2)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 32 | (2, 32, 32) | Z_{32} | 1 | 0 | 1 | C |
| 8 | 32 | (4, 4, 16) | $G(32, 20)$ | 1 | 0 | 1 | p-NA |
| 8 | 34 | (2, 17, 34) | Z_{34} | 1 | 0 | 1 | C |
| 8 | 36 | (2, 18, 18) | $Z_2 \times Z_{18}$ | 1 | 0 | 1 | A2 |
| 8 | 36 | (4, 4, 9) | $G(36, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 40 | (2, 10, 20) | $G(40, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 42 | (3, 6, 6) | $G(42, 1)$ | 0 | 1 | 1 | S-NA-NP |
| 8 | 42 | (3, 6, 6) | $G(42, 2)$ | 0 | 1 | 1 | S-NA-NP |
| 8 | 48 | (2, 6, 24) | $G(48, 25)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 48 | (2, 8, 12) | $G(48, 17)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 48 | (3, 4, 8) | $G(48, 28)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 60 | (2, 6, 10) | $G(60, 8)$ | 1 | 0 | 1 | S-NA-NP |

Table 6 Rotation and Tiling Groups, Genus 2-13 - part 6

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 8 | 64 | (2, 4, 32) | $G(64, 53)$ | 1 | 0 | 1 | p-NA |
| 8 | 72 | (2, 4, 18) | $G(72, 8)$ | 1 | 0 | 1 | S-NA-NP |
| 8 | 84 | (2, 6, 6) | $G(84, 7)$ | 0 | 1 | 1 | S-NA-NP |
| 8 | 168 | (3, 3, 4) | $G(168, 42)$ | 2 | 0 | 2 | NS |
| 8 | 336 | (2, 3, 8) | $G(336, 208)$ | 2 | 0 | 2 | NS |
| | | | | | | | |
| 9 | 19 | (19, 19, 19) | Z_{19} | 4 | 0 | 4 | C |
| 9 | 20 | (10, 20, 20) | Z_{20} | 2 | 0 | 2 | C |
| 9 | 21 | (7, 21, 21) | Z_{21} | 3 | 0 | 3 | C |
| 9 | 24 | (4, 24, 24) | Z_{24} | 2 | 0 | 2 | C |
| 9 | 24 | (6, 12, 12) | $Z_2 \times Z_{12}$ | 1 | 0 | 1 | A2 |
| 9 | 24 | (6, 8, 24) | Z_{24} | 1 | 0 | 1 | C |
| 9 | 24 | (8, 8, 12) | $G(24, 1)$ | 2 | 0 | 2 | S-NA-NP |
| 9 | 27 | (3, 27, 27) | Z_{27} | 1 | 0 | 1 | C |
| 9 | 28 | (4, 7, 28) | Z_{28} | 1 | 0 | 1 | C |
| 9 | 30 | (3, 10, 30) | Z_{30} | 1 | 0 | 1 | C |
| 9 | 32 | (4, 8, 8) | $Z_4 \times Z_8$ | 1 | 0 | 1 | A2 |
| 9 | 32 | (4, 8, 8) | $G(32, 4)$ | 1 | 0 | 1 | p-NA |
| 9 | 32 | (4, 8, 8) | $G(32, 5)$ | 1 | 0 | 1 | p-NA |
| 9 | 32 | (4, 8, 8) | $G(32, 8)$ | 1 | 0 | 1 | p-NA |
| 9 | 32 | (4, 8, 8) | $G(32, 12)$ | 2 | 0 | 2 | p-NA |
| 9 | 36 | (2, 36, 36) | Z_{36} | 1 | 0 | 1 | C |
| 9 | 36 | (4, 4, 18) | $G(36, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 38 | (2, 19, 38) | Z_{38} | 1 | 0 | 1 | C |
| 9 | 40 | (2, 20, 20) | $Z_2 \times Z_{20}$ | 1 | 0 | 1 | A2 |
| 9 | 40 | (4, 4, 10) | $G(40, 7)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 40 | (4, 4, 10) | $G(40, 12)$ | 0 | 1 | 1 | S-NA-NP |
| 9 | 42 | (2, 14, 21) | $G(42, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 48 | (2, 12, 12) | $G(48, 21)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 48 | (2, 12, 12) | $G(48, 31)$ | 2 | 0 | 2 | S-NA-NP |
| 9 | 48 | (2, 8, 24) | $G(48, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 48 | (2, 8, 24) | $G(48, 5)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 48 | (3, 4, 12) | $G(48, 31)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 48 | (3, 6, 6) | $G(48, 32)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 48 | (4, 4, 6) | $G(48, 19)$ | 1 | 0 | 1 | S-NA-NP |

Table 7 Rotation and Tiling Groups, Genus 2-13 - part 7

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|----------------|------|----------|-------|---------|
| 9 | 48 | (4, 4, 6) | $G(48, 30)$ | 2 | 0 | 2 | S-NA-NP |
| 9 | 48 | (4, 4, 6) | $G(48, 48)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 57 | (3, 3, 19) | $G(57, 1)$ | 0 | 1 | 1 | S-NA-NP |
| 9 | 60 | (3, 5, 5) | $G(60, 5)$ | 2 | 0 | 2 | NS |
| 9 | 64 | (2, 8, 8) | $G(64, 4)$ | 1 | 0 | 1 | p-NA |
| 9 | 64 | (2, 8, 8) | $G(64, 6)$ | 1 | 0 | 1 | p-NA |
| 9 | 64 | (2, 8, 8) | $G(64, 10)$ | 1 | 0 | 1 | p-NA |
| 9 | 64 | (2, 8, 8) | $G(64, 12)$ | 1 | 0 | 1 | p-NA |
| 9 | 64 | (2, 8, 8) | $G(64, 36)$ | 1 | 0 | 1 | p-NA |
| 9 | 64 | (4, 4, 4) | $G(64, 23)$ | 1 | 0 | 1 | p-NA |
| 9 | 64 | (4, 4, 4) | $G(64, 34)$ | 1 | 0 | 1 | p-NA |
| 9 | 64 | (4, 4, 4) | $G(64, 35)$ | 2 | 0 | 2 | p-NA |
| 9 | 72 | (2, 4, 36) | $G(72, 5)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 80 | (2, 4, 20) | $G(80, 14)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 96 | (2, 4, 12) | $G(96, 13)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 96 | (2, 4, 12) | $G(96, 186)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 96 | (2, 4, 12) | $G(96, 187)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 96 | (2, 6, 6) | $G(96, 70)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 96 | (3, 3, 6) | $G(96, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 96 | (3, 4, 4) | $G(96, 67)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 96 | (3, 4, 4) | $G(96, 227)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 120 | (2, 5, 6) | $G(120, 34)$ | 1 | 0 | 1 | NS |
| 9 | 120 | (2, 5, 6) | $G(120, 35)$ | 1 | 0 | 1 | NS |
| 9 | 128 | (2, 4, 8) | $G(128, 75)$ | 1 | 0 | 1 | p-NA |
| 9 | 128 | (2, 4, 8) | $G(128, 134)$ | 1 | 0 | 1 | p-NA |
| 9 | 128 | (2, 4, 8) | $G(128, 136)$ | 1 | 0 | 1 | p-NA |
| 9 | 128 | (2, 4, 8) | $G(128, 138)$ | 1 | 0 | 1 | p-NA |
| 9 | 160 | (2, 5, 5) | $G(160, 199)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 192 | (2, 3, 12) | $G(192, 194)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 192 | (2, 4, 6) | $G(192, 955)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 192 | (2, 4, 6) | $G(192, 990)$ | 1 | 0 | 1 | S-NA-NP |
| 9 | 320 | (2, 4, 5) | $G(320, 1582)$ | 1 | 0 | 1 | S-NA-NP |
| | | | | | | | |
| 10 | 21 | (21, 21, 21) | Z_{21} | 2 | 0 | 2 | C |
| 10 | 22 | (11, 22, 22) | Z_{22} | 5 | 0 | 5 | C |

Table 8 Rotation and Tiling Groups, Genus 2-13 - part 8

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 10 | 24 | (12, 12, 12) | $G(24, 11)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 24 | (6, 24, 24) | Z_{24} | 1 | 0 | 1 | C |
| 10 | 24 | (8, 12, 24) | Z_{24} | 2 | 0 | 2 | C |
| 10 | 25 | (5, 25, 25) | Z_{25} | 2 | 0 | 2 | C |
| 10 | 27 | (9, 9, 9) | $Z_3 \times Z_9$ | 1 | 0 | 1 | A2 |
| 10 | 27 | (9, 9, 9) | $G(27, 4)$ | 0 | 1 | 1 | p-NA |
| 10 | 28 | (4, 14, 28) | Z_{28} | 1 | 0 | 1 | C |
| 10 | 30 | (3, 30, 30) | Z_{30} | 1 | 0 | 1 | C |
| 10 | 30 | (5, 6, 30) | Z_{30} | 1 | 0 | 1 | C |
| 10 | 30 | (6, 6, 15) | $G(30, 2)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 33 | (3, 11, 33) | Z_{33} | 1 | 0 | 1 | C |
| 10 | 36 | (3, 12, 12) | $Z_3 \times Z_{12}$ | 1 | 0 | 1 | A2 |
| 10 | 36 | (3, 12, 12) | $G(36, 6)$ | 2 | 0 | 2 | S-NA-NP |
| 10 | 36 | (4, 6, 12) | $G(36, 6)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 36 | (6, 6, 6) | $Z_6 \times Z_6$ | 1 | 0 | 1 | A2 |
| 10 | 36 | (6, 6, 6) | $G(36, 12)$ | 2 | 0 | 2 | S-NA-NP |
| 10 | 40 | (2, 40, 40) | Z_{40} | 1 | 0 | 1 | C |
| 10 | 40 | (4, 4, 20) | $G(40, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 42 | (2, 21, 42) | Z_{42} | 1 | 0 | 1 | C |
| 10 | 42 | (3, 6, 14) | $G(42, 2)$ | 0 | 1 | 1 | S-NA-NP |
| 10 | 44 | (2, 22, 22) | $Z_2 \times Z_{22}$ | 1 | 0 | 1 | A2 |
| 10 | 44 | (4, 4, 11) | $G(44, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 48 | (2, 12, 24) | $G(48, 26)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 54 | (2, 9, 18) | $G(54, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 54 | (3, 6, 6) | $G(54, 5)$ | 2 | 0 | 2 | S-NA-NP |
| 10 | 54 | (3, 6, 6) | $G(54, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 54 | (3, 6, 6) | $G(54, 12)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 60 | (2, 6, 30) | $G(60, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 63 | (3, 3, 21) | $G(63, 3)$ | 0 | 1 | 1 | S-NA-NP |
| 10 | 72 | (2, 6, 12) | $G(72, 23)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 72 | (2, 6, 12) | $G(72, 28)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 72 | (2, 6, 12) | $G(72, 30)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 72 | (2, 8, 8) | $G(72, 39)$ | 0 | 1 | 1 | S-NA-NP |
| 10 | 72 | (3, 3, 12) | $G(72, 25)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 72 | (3, 4, 6) | $G(72, 42)$ | 1 | 0 | 1 | S-NA-NP |

Table 9 Rotation and Tiling Groups, Genus 2-13 - part 9

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 10 | 72 | (4, 4, 4) | $G(72, 41)$ | 0 | 2 | 2 | S-NA-NP |
| 10 | 80 | (2, 4, 40) | $G(80, 6)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 81 | (3, 3, 9) | $G(81, 7)$ | 1 | 0 | 1 | p-NA |
| 10 | 81 | (3, 3, 9) | $G(81, 9)$ | 1 | 0 | 1 | p-NA |
| 10 | 88 | (2, 4, 22) | $G(88, 7)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 108 | (2, 4, 12) | $G(108, 15)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 108 | (2, 6, 6) | $G(108, 17)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 108 | (2, 6, 6) | $G(108, 25)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 108 | (2, 6, 6) | $G(108, 38)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 108 | (3, 3, 6) | $G(108, 22)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 108 | (3, 4, 4) | $G(108, 15)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 108 | (3, 4, 4) | $G(108, 37)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 144 | (2, 3, 24) | $G(144, 122)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 144 | (2, 4, 8) | $G(144, 182)$ | 0 | 1 | 1 | S-NA-NP |
| 10 | 162 | (2, 3, 18) | $G(162, 14)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 168 | (2, 4, 7) | $G(168, 42)$ | 1 | 0 | 1 | NS |
| 10 | 180 | (2, 3, 15) | $G(180, 19)$ | 1 | 0 | 1 | NS |
| 10 | 216 | (2, 3, 12) | $G(216, 92)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 216 | (2, 4, 6) | $G(216, 87)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 216 | (2, 4, 6) | $G(216, 158)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 216 | (3, 3, 4) | $G(216, 153)$ | 1 | 1 | 2 | S-NA-NP |
| 10 | 324 | (2, 3, 9) | $G(324, 160)$ | 1 | 0 | 1 | S-NA-NP |
| 10 | 360 | (2, 4, 5) | $G(360, 118)$ | 1 | 0 | 1 | NS |
| 10 | 432 | (2, 3, 8) | $G(432, 734)$ | 0 | 1 | 1 | S-NA-NP |
| | | | | | | | |
| 11 | 23 | (23, 23, 23) | Z_{23} | 4 | 0 | 4 | C |
| 11 | 24 | (12, 24, 24) | Z_{24} | 2 | 0 | 2 | C |
| 11 | 30 | (6, 10, 15) | Z_{30} | 1 | 0 | 1 | C |
| 11 | 32 | (4, 16, 16) | $Z_2 \times Z_{16}$ | 1 | 0 | 1 | A2 |
| 11 | 32 | (4, 16, 16) | $G(32, 17)$ | 1 | 0 | 1 | p-NA |
| 11 | 32 | (8, 8, 8) | $G(32, 15)$ | 2 | 0 | 2 | p-NA |
| 11 | 33 | (3, 33, 33) | Z_{33} | 1 | 0 | 1 | C |
| 11 | 40 | (4, 8, 8) | $G(40, 3)$ | 0 | 2 | 2 | S-NA-NP |
| 11 | 44 | (2, 44, 44) | Z_{44} | 1 | 0 | 1 | C |
| 11 | 44 | (4, 4, 22) | $G(44, 1)$ | 1 | 0 | 1 | S-NA-NP |

Table 10 Rotation and Tiling Groups, Genus 2-13 - part 10

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 11 | 46 | (2, 23, 46) | Z_{46} | 1 | 0 | 1 | C |
| 11 | 48 | (2, 24, 24) | $Z_2 \times Z_{24}$ | 1 | 0 | 1 | A2 |
| 11 | 48 | (2, 24, 24) | $G(48, 24)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 48 | (3, 8, 8) | $G(48, 28)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 48 | (3, 8, 8) | $G(48, 29)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 48 | (4, 4, 12) | $G(48, 11)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 48 | (4, 4, 12) | $G(48, 12)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 48 | (4, 4, 12) | $G(48, 13)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 48 | (4, 6, 6) | $G(48, 32)$ | 3 | 0 | 3 | S-NA-NP |
| 11 | 60 | (2, 12, 12) | $G(60, 6)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 60 | (4, 4, 6) | $G(60, 7)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 64 | (2, 8, 16) | $G(64, 40)$ | 1 | 0 | 1 | p-NA |
| 11 | 64 | (2, 8, 16) | $G(64, 42)$ | 1 | 0 | 1 | p-NA |
| 11 | 66 | (2, 6, 33) | $G(66, 2)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 80 | (2, 8, 8) | $G(80, 28)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 80 | (2, 8, 8) | $G(80, 29)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 80 | (4, 4, 4) | $G(80, 30)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 80 | (4, 4, 4) | $G(80, 31)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 88 | (2, 4, 44) | $G(88, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 96 | (2, 4, 24) | $G(96, 28)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 96 | (2, 4, 24) | $G(96, 32)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 96 | (2, 6, 8) | $G(96, 189)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 96 | (2, 6, 8) | $G(96, 190)$ | 1 | 0 | 1 | S-NA-NP |
| 11 | 120 | (2, 4, 12) | $G(120, 36)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 120 | (2, 6, 6) | $G(120, 34)$ | 1 | 0 | 1 | NS |
| 11 | 120 | (3, 4, 4) | $G(120, 34)$ | 1 | 0 | 1 | NS |
| 11 | 160 | (2, 4, 8) | $G(160, 82)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 160 | (2, 4, 8) | $G(160, 85)$ | 0 | 1 | 1 | S-NA-NP |
| 11 | 240 | (2, 4, 6) | $G(240, 189)$ | 1 | 0 | 1 | NS |
| | | | | | | | |
| 12 | 25 | (25, 25, 25) | Z_{25} | 3 | 0 | 3 | C |
| 12 | 26 | (13, 26, 26) | Z_{26} | 6 | 0 | 6 | C |
| 12 | 27 | (9, 27, 27) | Z_{27} | 3 | 0 | 3 | C |
| 12 | 28 | (14, 14, 14) | $Z_2 \times Z_{14}$ | 2 | 0 | 2 | A2 |
| 12 | 28 | (7, 28, 28) | Z_{28} | 3 | 0 | 3 | C |

Table 11 Rotation and Tiling Groups, Genus 2-13 - part 11

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 12 | 30 | (10, 10, 15) | $G(30, 1)$ | 2 | 0 | 2 | S-NA-NP |
| 12 | 30 | (5, 30, 30) | Z_{30} | 2 | 0 | 2 | C |
| 12 | 30 | (6, 15, 30) | Z_{30} | 1 | 0 | 1 | C |
| 12 | 32 | (4, 32, 32) | Z_{32} | 1 | 0 | 1 | C |
| 12 | 35 | (5, 7, 35) | Z_{35} | 1 | 0 | 1 | C |
| 12 | 36 | (3, 36, 36) | Z_{36} | 1 | 0 | 1 | C |
| 12 | 36 | (4, 9, 36) | Z_{36} | 1 | 0 | 1 | C |
| 12 | 36 | (6, 9, 9) | $G(36, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 39 | (3, 13, 39) | Z_{39} | 1 | 0 | 1 | C |
| 12 | 40 | (4, 10, 10) | $G(40, 10)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 40 | (5, 8, 8) | $G(40, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 40 | (5, 8, 8) | $G(40, 3)$ | 0 | 1 | 1 | S-NA-NP |
| 12 | 42 | (3, 14, 14) | $G(42, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 42 | (6, 6, 7) | $G(42, 1)$ | 0 | 1 | 1 | S-NA-NP |
| 12 | 42 | (6, 6, 7) | $G(42, 2)$ | 0 | 1 | 1 | S-NA-NP |
| 12 | 42 | (6, 6, 7) | $G(42, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 48 | (2, 48, 48) | Z_{48} | 1 | 0 | 1 | C |
| 12 | 48 | (4, 4, 24) | $G(48, 8)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 48 | (4, 6, 8) | $G(48, 16)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 48 | (4, 6, 8) | $G(48, 28)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 50 | (2, 25, 50) | Z_{50} | 1 | 0 | 1 | C |
| 12 | 52 | (2, 26, 26) | $Z_2 \times Z_{26}$ | 1 | 0 | 1 | A2 |
| 12 | 52 | (4, 4, 13) | $G(52, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 52 | (4, 4, 13) | $G(52, 3)$ | 0 | 1 | 1 | S-NA-NP |
| 12 | 55 | (5, 5, 5) | $G(55, 1)$ | 0 | 2 | 2 | S-NA-NP |
| 12 | 56 | (2, 14, 28) | $G(56, 9)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 60 | (2, 10, 30) | $G(60, 11)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 60 | (2, 15, 15) | $G(60, 9)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 80 | (2, 8, 10) | $G(80, 15)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 84 | (2, 6, 14) | $G(84, 8)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 84 | (3, 3, 14) | $G(84, 11)$ | 0 | 1 | 1 | S-NA-NP |
| 12 | 96 | (2, 4, 48) | $G(96, 7)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 104 | (2, 4, 26) | $G(104, 8)$ | 1 | 0 | 1 | S-NA-NP |
| 12 | 110 | (2, 5, 10) | $G(110, 1)$ | 0 | 2 | 2 | S-NA-NP |
| 12 | 120 | (2, 4, 15) | $G(120, 38)$ | 1 | 0 | 1 | S-NA-NP |

Table 12 Rotation and Tiling Groups, Genus 2-13 - part 12

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|--------------|---------------------|------|----------|-------|---------|
| 13 | 27 | (27, 27, 27) | Z_{27} | 2 | 0 | 2 | C |
| 13 | 28 | (14, 28, 28) | Z_{28} | 3 | 0 | 3 | C |
| 13 | 30 | (10, 15, 30) | Z_{30} | 3 | 0 | 3 | C |
| 13 | 32 | (8, 16, 16) | $Z_2 \times Z_{16}$ | 2 | 0 | 2 | A2 |
| 13 | 32 | (8, 16, 16) | $G(32, 17)$ | 2 | 0 | 2 | p-NA |
| 13 | 36 | (4, 18, 36) | Z_{36} | 1 | 0 | 1 | C |
| 13 | 36 | (6, 12, 12) | $Z_3 \times Z_{12}$ | 1 | 0 | 1 | A2 |
| 13 | 36 | (6, 12, 12) | $G(36, 6)$ | 2 | 0 | 2 | S-NA-NP |
| 13 | 36 | (9, 9, 9) | $G(36, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 39 | (3, 39, 39) | Z_{39} | 1 | 0 | 1 | C |
| 13 | 40 | (4, 10, 20) | $Z_2 \times Z_{20}$ | 1 | 0 | 1 | A2 |
| 13 | 42 | (3, 14, 42) | Z_{42} | 1 | 0 | 1 | C |
| 13 | 45 | (3, 15, 15) | $Z_3 \times Z_{15}$ | 1 | 0 | 1 | A2 |
| 13 | 48 | (3, 12, 12) | $G(48, 31)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 48 | (3, 12, 12) | $G(48, 33)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 48 | (4, 6, 12) | $G(48, 21)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 48 | (4, 6, 12) | $G(48, 31)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 48 | (6, 6, 6) | $G(48, 32)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 52 | (2, 52, 52) | Z_{52} | 1 | 0 | 1 | C |
| 13 | 52 | (4, 4, 26) | $G(52, 1)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 54 | (2, 27, 54) | Z_{54} | 1 | 0 | 1 | C |
| 13 | 56 | (2, 28, 28) | $Z_2 \times Z_{28}$ | 1 | 0 | 1 | A2 |
| 13 | 56 | (4, 4, 14) | $G(56, 6)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 60 | (5, 5, 5) | $G(60, 5)$ | 1 | 0 | 1 | NS |
| 13 | 64 | (2, 16, 16) | $G(64, 29)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (2, 16, 16) | $G(64, 30)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (2, 16, 16) | $G(64, 31)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (4, 4, 8) | $G(64, 8)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (4, 4, 8) | $G(64, 9)$ | 2 | 0 | 2 | p-NA |
| 13 | 64 | (4, 4, 8) | $G(64, 18)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (4, 4, 8) | $G(64, 20)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (4, 4, 8) | $G(64, 21)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (4, 4, 8) | $G(64, 32)$ | 1 | 0 | 1 | p-NA |
| 13 | 64 | (4, 4, 8) | $G(64, 33)$ | 2 | 0 | 2 | p-NA |
| 13 | 72 | (2, 12, 12) | $G(72, 21)$ | 1 | 0 | 1 | S-NA-NP |

Table 13 Rotation and Tiling Groups, Genus 2-13 - part 13

| σ | $ G $ | (l, m, n) | Group | #Kal | #non-Kal | total | Type |
|----------|-------|-------------|----------------|------|----------|-------|---------|
| 13 | 72 | (2, 12, 12) | $G(72, 27)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 72 | (2, 9, 18) | $G(72, 16)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 72 | (3, 4, 12) | $G(72, 42)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 72 | (3, 6, 6) | $G(72, 44)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 72 | (3, 6, 6) | $G(72, 47)$ | 2 | 0 | 2 | S-NA-NP |
| 13 | 72 | (4, 4, 6) | $G(72, 45)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 78 | (2, 6, 39) | $G(78, 4)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 90 | (2, 6, 15) | $G(90, 7)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 96 | (3, 4, 6) | $G(96, 3)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 96 | (3, 4, 6) | $G(96, 68)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 96 | (3, 4, 6) | $G(96, 70)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 96 | (3, 4, 6) | $G(96, 71)$ | 0 | 1 | 1 | S-NA-NP |
| 13 | 104 | (2, 4, 52) | $G(104, 5)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 112 | (2, 4, 28) | $G(112, 13)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 120 | (2, 5, 10) | $G(120, 35)$ | 1 | 0 | 1 | NS |
| 13 | 128 | (2, 4, 16) | $G(128, 71)$ | 1 | 0 | 1 | p-NA |
| 13 | 128 | (2, 4, 16) | $G(128, 79)$ | 1 | 0 | 1 | p-NA |
| 13 | 144 | (2, 4, 12) | $G(144, 115)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 144 | (3, 3, 6) | $G(144, 184)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 180 | (3, 3, 5) | $G(180, 19)$ | 1 | 0 | 1 | NS |
| 13 | 288 | (2, 3, 12) | $G(288, 1024)$ | 1 | 0 | 1 | S-NA-NP |
| 13 | 360 | (2, 3, 10) | $G(360, 121)$ | 1 | 0 | 1 | NS |

References

- [1] Rose-Hulman NSF-REU Tilings web site,
<http://www.tilings.org/index.html>.