Kaleidoscopic Tilings on Surfaces

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1. Overview of Notes

Introduction The goal of these notes is to provide a quick introduction to tilings on Riemann surfaces and their associated tiling groups, and then to present lists of tiling problems suitable for undergraduate research problems. Additional chapters provide further information on a need-to-know basis. The tilings start off as interesting geometrical objects which are easily visualized in the simple and well-understood cases. The tiling group of symmetries of the tiling is easily defined and understood in these simple cases. In higher genus cases where the geometry is unknown and visualization is not possible, the tiling group is needed to completely capture the geometry of the tilings. We only need to get the tiling group to tell us the right story. By using computer software to perform elementary group theoretic, combinatorial and other calculations, it is easy to work out numerous examples of tiling groups and the geometry of the tilings. Using these examples we can discover, formulate and test conjectures, about tilings on surfaces and then eventually prove theorems.

Background, problems, more background We develop our material in the following order. In Chapter 2 we introduce surfaces, tilings, their associated geometry, and the tiling group. This will give us enough background to understand the problems given in Chapter 3. The problems are of two types: "warm up" problems, and REU problems. The warm up problems form a partial list of known results that give a good preparation for the REU problems, particularly for translating geometric problems into group theory problems. They should be briefly read and understood before working on the REU problems. Working out one or two of the warm up problems on an as needed basis will provide ideas for solving the REU problems. The REU problems are the suggested problems for the research program. In all of the REU problems partial results or compiling and classifying lists of interesting examples would be already be a significant contribution. The last three sections of Chapter 3 introduce a few further concepts, results, examples and additional problems. Each technical report by REU participants has a section called further research problems giving additional problems to be worked on.

Chapters 4 - 8 expand on the introductory material:

- Chapter 4 reviews some basic background on hyperbolic geometry, (background reference [10])
- Chapter 5 gives some background information on groups, group actions and some constructions of some finite groups,
- Chapter 6 discusses the topology of orbit spaces, fundamental groups, and covering spaces, (background references [24], [17])
- Chapter 7 discusses the homology and cohomology of surfaces,(background references [24], [17])

- Chapter 8 briefly discuses the idea of a Hecke algebra, and
- Chapter 9 briefly discusses the Teichmüller and moduli spaces.

These chapters yield some more sophisticated concepts and tools that may be useful in attacking some of the problems but are not necessary in the initial steps. Some further REU problems requiring more background are listed in Chapters 7,8, and 9

Assumed algebraic background and computer tools The only background assumed is a standard junior level courses in abstract algebra with a strong emphasis on group theory. The principal tool of analysis is Magma. This software tool is a computer algebra system for groups and discrete mathematics. Just think of it as Maple or Mathematica for group theory (though the interface is not as nice). Though Magma is the tool of choice for the group-theoretic calculations, Maple, Mathematica and Matlab all come in handy for geometric illustration and symbolic computation. Additional background reference are noted in the lsit above. The can be learned on a need to know basis.

History and Resources The Rose-Hulman REU has been in existence since the summer of 1989. The first years concentrated on topics in computational group theory. In 1996 the tiling problems were introduced and have been worked on by a portion of the participants each summer. The results and resources developed for the tilings program are available at the following web location:

http://www.tilings.org/